Target Response Adaptation for Correlation Filter Tracking

Adel Bibi, Matthias Mueller, and Bernard Ghanem

Image and Video Understanding Laboratory (IVUL),
King Abdullah University of Science and Technology (KAUST), Saudi Arabia
{adel.bibi,matthias.mueller.2,bernard.ghanem}@kaust.edu.sa

In this supplementary material, we followup our discussion on the solution to the following problem:

$$\min_{\mathbf{w},\mathbf{y}} \quad ||\mathbf{A}\mathbf{w}-\mathbf{y}||_2^2 + \lambda_1||\mathbf{w}||_2^2 + \lambda_2||\mathbf{y}-\mathbf{y}_0||_2^2 \tag{1}$$

The problem is convex quadratic and a stationary point is necessary and sufficient for global optimality. The following sections will discuss how Problem 1 is solved in the primal domain, dual domain, and for both single and multiple templates along with the formula used to generate the response map, whose maximum value determines the current detection. Lastly, we discuss a one way of incorporating SRDCF [2] with our proposed target adaptive framework.

1 Solution to Problem 1 in the Primal Domain

1.1 Using a Single Template

Problem 1 can be rewritten in terms of $\mathbf{z}$ with $\mathbf{z}^T = \begin{bmatrix}\mathbf{w}^T \ \mathbf{y}^T\end{bmatrix}$:

$$f(\mathbf{z}) = ||\begin{bmatrix}\mathbf{A} & -\mathbf{I}\end{bmatrix}\mathbf{z}||_2^2 + \lambda_1||\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}\mathbf{z}||_2^2 + \lambda_2||\begin{bmatrix}\mathbf{0} & \mathbf{I}\end{bmatrix}\mathbf{z}-\mathbf{y}_0||_2^2 \quad , \tag{2}$$

where $\mathbf{w} \in \mathbb{R}^n$, $\mathbf{A} \in \mathbb{R}^{n\times n}$, $\mathbf{y} \in \mathbb{R}^n$, $\mathbf{y}_0 \in \mathbb{R}^n$, $\mathbf{z} \in \mathbb{R}^{2n}$, then:

$$\nabla_{\mathbf{z}} f(\mathbf{z}) = \begin{bmatrix}\mathbf{A}^T\mathbf{A} & -\mathbf{A}^T \\ -\mathbf{A} & \mathbf{I}\end{bmatrix}\mathbf{z} + \lambda_1\begin{bmatrix}\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0}\end{bmatrix}\mathbf{z} + \lambda_2\begin{bmatrix}\mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}\end{bmatrix}\mathbf{z} - \lambda_2\begin{bmatrix}\mathbf{0} \\ \mathbf{I}\end{bmatrix}\mathbf{y}_0 = 0$$

$$\nabla_{\mathbf{z}} f(\mathbf{z}) = \begin{bmatrix}\mathbf{A}^T\mathbf{A}+\lambda_1\mathbf{I} & -\mathbf{A}^T \\ -\mathbf{A} & (1+\lambda_2)\mathbf{I}\end{bmatrix}\mathbf{z} = \lambda_2\begin{bmatrix}\mathbf{0} \\ \mathbf{I}\end{bmatrix}\mathbf{y}_0$$

$$\begin{bmatrix}\mathbf{F} & \mathbf{0} \\ \mathbf{0} & \mathbf{F}\end{bmatrix}\begin{bmatrix}diag(\hat{\mathbf{a}}_1 \odot \hat{\mathbf{a}}_1^* + \lambda_1) & -diag(\hat{\mathbf{a}}_1^*) \\ -diag(\hat{\mathbf{a}}_1) & diag(1+\lambda_2)\end{bmatrix}\begin{bmatrix}\mathbf{F}^H & \mathbf{0} \\ \mathbf{0} & \mathbf{F}^H\end{bmatrix}\mathbf{z} = \lambda_2\begin{bmatrix}\mathbf{0} \\ \mathbf{I}\end{bmatrix}\mathbf{y}_0$$

$$\begin{bmatrix}diag(\hat{\mathbf{a}}_1 \odot \hat{\mathbf{a}}_1^* + \lambda_1) & -diag(\hat{\mathbf{a}}_1^*) \\ -diag(\hat{\mathbf{a}}_1) & diag(1+\lambda_2)\end{bmatrix}\begin{bmatrix}\hat{\mathbf{w}}^* \\ \hat{\mathbf{y}}^*\end{bmatrix} = \lambda_2\begin{bmatrix}\mathbf{0} \\ \mathbf{F}^H\end{bmatrix}\mathbf{y}_0$$

$$\begin{bmatrix}\hat{\mathbf{w}}^* \\ \hat{\mathbf{y}}^*\end{bmatrix} = \lambda_2\begin{bmatrix}diag(\hat{\mathbf{a}}_1 \odot \hat{\mathbf{a}}_1^* + \lambda_1) & -diag(\hat{\mathbf{a}}_1^*) \\ -diag(\hat{\mathbf{a}}_1) & diag(1+\lambda_2)\end{bmatrix}^{-1}\begin{bmatrix}\mathbf{0} \\ \mathbf{F}^H\end{bmatrix}\mathbf{y}_0$$

Note that the inverse lemma states:

$$\begin{bmatrix}\mathbf{B} & \mathbf{N}\\ \mathbf{V} & \mathbf{C}\end{bmatrix}^{-1} = \begin{bmatrix}(\mathbf{B}-\mathbf{N}\mathbf{C}^{-1}\mathbf{V})^{-1} & -(\mathbf{B}-\mathbf{N}\mathbf{C}^{-1}\mathbf{V})^{-1}\mathbf{N}\mathbf{C}^{-1}\\ -\mathbf{C}^{-1}\mathbf{V}(\mathbf{B}-\mathbf{N}\mathbf{C}^{-1}\mathbf{V})^{-1} & \mathbf{C}^{-1}\mathbf{V}(\mathbf{B}-\mathbf{N}\mathbf{C}^{-1}\mathbf{V})^{-1}\mathbf{N}\mathbf{C}^{-1}+\mathbf{C}^{-1}\end{bmatrix} \tag{3}$$

Then:

$$\begin{aligned}
\mathbf{B}-\mathbf{N}\mathbf{C}^{-1}\mathbf{V} &= diag(\hat{\mathbf{a}}_1 \odot \hat{\mathbf{a}}_1^* + \lambda_1) - diag(\hat{\mathbf{a}}_1^*)diag^{-1}(1+\lambda_2)diag(\hat{\mathbf{a}}_1)\\
(\mathbf{B}-\mathbf{N}\mathbf{C}^{-1}\mathbf{V})^{-1} &= diag\Big(\frac{1+\lambda_2}{\lambda_2(\hat{\mathbf{a}}_1 \odot \hat{\mathbf{a}}_1^*) + \lambda_1(1+\lambda_2)}\Big)\\
-(\mathbf{B}-\mathbf{N}\mathbf{C}^{-1}\mathbf{V})^{-1}\mathbf{N}\mathbf{C}^{-1} &= diag\Big(\frac{\hat{\mathbf{a}}_1^*}{\lambda_2(\hat{\mathbf{a}}_1 \odot \hat{\mathbf{a}}_1^*) + \lambda_1(1+\lambda_2)}\Big)
\end{aligned}$$

Since:

$$\hat{\mathbf{w}}^* = -\lambda_2(\mathbf{B}-\mathbf{N}\mathbf{C}^{-1}\mathbf{V})^{-1}\mathbf{N}\mathbf{C}^{-1}\mathbf{F}^H\mathbf{y}_o \tag{4}$$

Then:

$$\hat{\mathbf{w}}^* = \frac{\lambda_2(\hat{\mathbf{a}}_1^* \odot \hat{\mathbf{y}}_o^*)}{\lambda_2(\hat{\mathbf{a}}_1 \odot \hat{\mathbf{a}}_1^*) + \lambda_1(1+\lambda_2)} \Rightarrow \quad \hat{\mathbf{w}} = \frac{\lambda_2(\hat{\mathbf{a}}_1 \odot \hat{\mathbf{y}}_o)}{\lambda_2(\hat{\mathbf{a}}_1 \odot \hat{\mathbf{a}}_1^*) + \lambda_1(1+\lambda_2)} \tag{5}$$

Detection Formula with a Single Template

As for the detection formula in the primal domain, we consider a new test sample $\mathbf{u}$. For detection, we construct all the circular shifts of $\mathbf{u}$ in matrix $\mathbf{U}$. Therefore, the response map on the test sample is:

$$\mathbf{T}(\mathbf{u}) = \mathbf{U}\mathbf{w} = \mathbf{F}diag(\hat{\mathbf{u}})\mathbf{F}^H\mathbf{w} \Rightarrow \hat{\mathbf{T}}^*(\mathbf{u}) = \hat{\mathbf{u}} \odot \hat{\mathbf{w}}^* \quad \Rightarrow \hat{\mathbf{T}}(\mathbf{u}) = \hat{\mathbf{u}}^* \odot \hat{\mathbf{w}} \tag{6}$$

1.2 Using Multiple Templates

Following is the derivation of the solution for the multiple template case in primal domain, i.e. when $\tilde{\mathbf{A}} \in \mathbb{R}^{kn\times n}$, k is the total number of templates, and $\tilde{\mathbf{A}}^T = \begin{bmatrix}\mathbf{A}_1^T & \mathbf{A}_2^T & \dots & \mathbf{A}_k^T\end{bmatrix}$ and $\tilde{\mathbf{I}}^T = \begin{bmatrix}\mathbf{I}_1 & \mathbf{I}_2 & \dots & \mathbf{I}_k\end{bmatrix}$. Then:

$$f(\mathbf{z}) = ||\begin{bmatrix}\tilde{\mathbf{A}} & -\tilde{\mathbf{I}}\end{bmatrix}\mathbf{z}||_2^2 + \lambda_1||\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}\mathbf{z}||_2^2 + \lambda_2||\begin{bmatrix}\mathbf{0} & \mathbf{I}\end{bmatrix}\mathbf{z} - \mathbf{y}_0||_2^2$$

$$\begin{aligned}
\nabla f(\mathbf{z}) &= \begin{bmatrix}\tilde{\mathbf{A}}^T\tilde{\mathbf{A}} & -\tilde{\mathbf{A}}^T\tilde{\mathbf{I}}\\ -\tilde{\mathbf{I}}^T\tilde{\mathbf{A}} & \tilde{\mathbf{I}}^T\tilde{\mathbf{I}}\end{bmatrix}\mathbf{z} + \lambda_1\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\mathbf{z} + \lambda_2\begin{bmatrix}\mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}\end{bmatrix}\mathbf{z} - \lambda_2\begin{bmatrix}\mathbf{0}\\ \mathbf{I}\end{bmatrix}\mathbf{y}_0 = 0\\
\nabla f(\mathbf{z}) &= \underbrace{\begin{bmatrix}\tilde{\mathbf{A}}^T\tilde{\mathbf{A}} + \lambda_1\mathbf{I} & -\tilde{\mathbf{A}}^T\tilde{\mathbf{I}}\\ -\tilde{\mathbf{I}}^T\tilde{\mathbf{A}} & (k+\lambda_2)\mathbf{I}\end{bmatrix}}_{\Gamma}\mathbf{z} = \lambda_2\begin{bmatrix}\mathbf{0}\\ \mathbf{I}\end{bmatrix}\mathbf{y}_0
\end{aligned}$$

Now, note the following: $\tilde{\mathbf{A}}^T\tilde{\mathbf{A}} = \sum_{i=1}^{k}\mathbf{A}_i^T\mathbf{A}_i$, $\tilde{\mathbf{I}}^T\tilde{\mathbf{I}} = \sum_{i=1}^{k}\mathbf{I}_i^T\mathbf{I}_i = k\mathbf{I}$, and $\tilde{\mathbf{A}}^T\tilde{\mathbf{I}} = \sum_{i=1}^{k}\mathbf{A}_i^T$, and that $\tilde{\mathbf{I}}^T\tilde{\mathbf{A}} = \sum_{i=1}^{k}\mathbf{A}_i$. It is clear that $\tilde{\mathbf{A}}^T\tilde{\mathbf{A}}$ and $\tilde{\mathbf{I}}^T\tilde{\mathbf{I}}$ are circulant which is a sum of circulant matrices. Therefore, the matrix Γ is block wise circulant such that $\Gamma \in \mathbb{R}^{2n\times 2n}$. Similar to the single template case, we have:

$$\begin{bmatrix}\tilde{\mathbf{w}}^*\\ \tilde{\mathbf{y}}^*\end{bmatrix} = \lambda_2 \begin{bmatrix}\sum_{i=1}^{k} diag(\hat{\mathbf{a}}_{1i}\odot\hat{\mathbf{a}}_{1i}^* + \frac{\lambda_1}{k}) & -\sum_{i=1}^{k} diag(\hat{\mathbf{a}}_{1i}^*)\\ -\sum_{i=1}^{k} diag(\hat{\mathbf{a}}_{1i}) & diag(k+\lambda_2)\end{bmatrix}^{-1}\begin{bmatrix}\mathbf{0}\\ \mathbf{F^H}\end{bmatrix}\mathbf{y}_0 \tag{7}$$

Using the inverse lemma in Eq 3 on Γ, we get:

$$\mathbf{B}-\mathbf{N}\mathbf{C}^{-1}\mathbf{V} = diag(\sum_i^k(\hat{\mathbf{a}}_{1i}\odot\hat{\mathbf{a}}_{1i})+\lambda_1) - \frac{diag(\sum_i^k\hat{\mathbf{a}}_{1i}^*)diag(\sum_i^k(\hat{\mathbf{a}}_{1i}))}{(k+\lambda_2)} \tag{8}$$

$$\begin{aligned}&-(\mathbf{B}-\mathbf{N}\mathbf{C}^{-1}\mathbf{V})^{-1}\mathbf{N}\mathbf{C}^{-1} =\\ &diag\left(\frac{\sum_i^k\hat{\mathbf{a}}_{1i}^*}{(k+\lambda_2)(\sum_i^k\hat{\mathbf{a}}_{1i}\odot\hat{\mathbf{a}_{1i}})+(k+\lambda_2)\lambda_1-(\sum_i^k\hat{\mathbf{a}}_{1i}^*\odot\sum_i^k\hat{\mathbf{a}}_{1i})}\right)\end{aligned} \tag{9}$$

Then:

$$\hat{\mathbf{w}}^* = \frac{\lambda_2(\sum_i^k\hat{\mathbf{a}_{1i}^*})\odot\hat{\mathbf{y}}_o^*}{(k+\lambda_2)(\sum_i^k\hat{\mathbf{a}}_{1i}\odot\hat{\mathbf{a}_{1i}})+(k+\lambda_2)\lambda_1-(\sum_i^k\hat{\mathbf{a}}_{1i}^*\odot\sum_i^k\hat{\mathbf{a}}_{1i})} \tag{10}$$

It is to be noted that when $k=1$, then Eq 10 reduces to the single template case in Eq 5, since $\sum_i^k(\hat{\mathbf{a}}_{1i}^*\odot\hat{\mathbf{a}}_{1i}) = (\sum_i^k\hat{\mathbf{a}}_{1i}^*\odot\sum_i^k\hat{\mathbf{a}}_{1i})$ for $k=1$.

Detection Formula with Multiple Templates

Here, the detection formula is similar to Eq 6.

2 Solution to Problem 1 in the Dual Domain

2.1 Using a Single Template

The optimization problem becomes:

$$f(\mathbf{z}) = ||\begin{bmatrix}\mathbf{A} & -\mathbf{I}\end{bmatrix}\mathbf{z}||_2^2 + \lambda_1||\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}\mathbf{z}||_2^2 + \lambda_2||\begin{bmatrix}\mathbf{0} & \mathbf{I}\end{bmatrix}\mathbf{z}-\mathbf{y}_0||_2^2 \tag{11}$$

For simpler notation, let $\mathbf{G} = \begin{bmatrix}\mathbf{A} & -\mathbf{I}\end{bmatrix}$, also let $\mathbf{E} = \begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}$, and $\mathbf{D} = \begin{bmatrix}\mathbf{0} & \mathbf{I}\end{bmatrix}$

$$\begin{aligned}f(\mathbf{z}) &= ||\mathbf{G}\mathbf{z}||_2^2 + \lambda_1||\mathbf{E}\mathbf{z}||_2^2 + \lambda_2||\mathbf{D}\mathbf{z}-\mathbf{y}_0||_2^2\\ &= \lambda_2\sum_i\left(\mathbf{z}^T\mathbf{d}_i - y_{0i}\right)^2 + \lambda_1||\mathbf{E}\mathbf{z}||_2^2 + ||\mathbf{G}\mathbf{z}||_2^2\end{aligned} \tag{12}$$

Then:

$$\nabla_{\mathbf{z}} f(\mathbf{z}) = 2\lambda_2 \sum_i \left(\mathbf{z}^T\mathbf{d}_i - y_{0i}\right)\mathbf{d}_i + 2\lambda_1 \mathbf{E}^T\mathbf{E}\mathbf{z} + 2\mathbf{G}^T\mathbf{G}\mathbf{z} = 0$$

$$\lambda_2 \sum_i \left(\mathbf{z}^T\mathbf{d}_i - y_{0i}\right)\mathbf{d}_i = -\left(\lambda_1\mathbf{E}^T\mathbf{E} + \mathbf{G}^T\mathbf{G}\right)\mathbf{z}$$

$$\mathbf{z} = -\lambda_2\left(\lambda_1\mathbf{E}^T\mathbf{E} + \mathbf{G}^T\mathbf{G}\right)^{-1}\sum_i \left(\mathbf{z}^T\mathbf{d}_i - y_{0i}\right)\mathbf{d}_i$$

Let $\alpha_i = -\lambda_2\left(\mathbf{z}^T\mathbf{d}_i - y_{0i}\right)$. Then $\mathbf{z} = \left(\lambda_1\mathbf{E}^T\mathbf{E} + \mathbf{G}^T\mathbf{G}\right)^{-1}\mathbf{D}^T\alpha$. Let $\mathbf{K} = \left(\lambda_1\mathbf{E}^T\mathbf{E} + \mathbf{G}^T\mathbf{G}\right)$.
By substituting the dual variables α into Eq 12, we obtain:

$$f(\alpha) = \lambda_2 \sum_i \left(\alpha^T\mathbf{D}\mathbf{K}^{-1}\mathbf{d}_i - y_{0i}\right)^2 + \lambda_1||\mathbf{E}\mathbf{K}^{-1}\mathbf{D}^T\alpha||_2^2 + ||\mathbf{G}\mathbf{K}^{-1}\mathbf{D}^T\alpha||_2^2 \tag{13}$$

Then, the new dual objective is given by:

$$f(\alpha) = \lambda_2||\mathbf{D}\mathbf{K}^{-1}\mathbf{D}^T\alpha - \mathbf{y}_0||_2^2 + \lambda_1||\mathbf{E}\mathbf{K}^{-1}\mathbf{D}^T\alpha||_2^2 + ||\mathbf{G}\mathbf{K}^{-1}\mathbf{D}^T\alpha||_2^2 \tag{14}$$

By setting the gradient to zero, the solution to Problem 14 is obtained by solving the following linear system:

$$\left(\lambda_2\mathbf{D}\mathbf{K}^{-1}\mathbf{D}^T\mathbf{D}\mathbf{K}^{-1}\mathbf{D}^T + \lambda_1\mathbf{D}\mathbf{K}^{-1}\mathbf{E}^T\mathbf{E}\mathbf{K}^{-1}\mathbf{D}^T + \mathbf{D}\mathbf{K}^{-1}\mathbf{G}^T\mathbf{G}\mathbf{K}^{-1}\mathbf{D}^T\right)\alpha = \lambda_2\mathbf{D}\mathbf{K}^{-1}\mathbf{D}^T\mathbf{y}_0$$

$$\mathbf{D}\mathbf{K}^{-1}\underbrace{\left(\lambda_2\mathbf{D}^T\mathbf{D} + \lambda_1\mathbf{E}^T\mathbf{E} + \mathbf{G}^T\mathbf{G}\right)}_{\Psi}\mathbf{K}^{-1}\mathbf{D}^T\alpha = \lambda_2\mathbf{D}\mathbf{K}^{-1}\mathbf{D}^T\mathbf{y}_0 \tag{15}$$

$$\mathbf{D}\begin{bmatrix}(\mathbf{A}^T\mathbf{A} + \lambda_1\mathbf{I}) & -\mathbf{A}^T \\ -\mathbf{A} & \mathbf{I}\end{bmatrix}^{-1}\underbrace{\begin{bmatrix}(\mathbf{A}^T\mathbf{A} + \lambda_1\mathbf{I}) & -\mathbf{A}^T \\ -\mathbf{A} & (1+\lambda_2)\mathbf{I}\end{bmatrix}}_{\Psi}\begin{bmatrix}(\mathbf{A}^T\mathbf{A} + \lambda_1\mathbf{I}) & -\mathbf{A}^T \\ -\mathbf{A} & \mathbf{I}\end{bmatrix}^{-1}$$
$$\mathbf{D}^T\alpha = \lambda_2\mathbf{D}\begin{bmatrix}(\mathbf{A}^T\mathbf{A} + \lambda_1\mathbf{I}) & -\mathbf{A}^T \\ -\mathbf{A} & \mathbf{I}\end{bmatrix}^{-1}\mathbf{D}^T\mathbf{y}_0 \tag{16}$$

By using the inverse lemma and the diagonalization properties of circulant matrices (similar to inverting the circulant diagonal matrix in the single template case), one can show the following:

$$\begin{aligned}\mathbf{K} &= \begin{bmatrix}(\mathbf{A}^T\mathbf{A}+\lambda_1\mathbf{I}) & -\mathbf{A}^T \\ -\mathbf{A} & \mathbf{I}\end{bmatrix} \\ \mathbf{K}^{-1} &= \begin{bmatrix}\frac{1}{\lambda_1}\mathbf{I} & \frac{1}{\lambda_1}\mathbf{A}^T \\ \frac{1}{\lambda_1}\mathbf{A} & \frac{1}{\lambda_1}\mathbf{A}\mathbf{A}^T+\mathbf{I}\end{bmatrix}\end{aligned} \tag{17}$$

Then:

$$\begin{aligned}\mathbf{K}^{-1}\Psi\mathbf{K}^{-1} &= \begin{bmatrix}\mathbf{I} & \frac{\lambda_2}{\lambda_1}\mathbf{A}^T \\ \mathbf{0} & \frac{\lambda_2}{\lambda_1}\mathbf{A}\mathbf{A}^T+(1+\lambda_2)\mathbf{I}\end{bmatrix}\begin{bmatrix}\frac{1}{\lambda_1}\mathbf{I} & \frac{1}{\lambda_1}\mathbf{A}^T \\ \frac{1}{\lambda_1}\mathbf{A} & \frac{1}{\lambda_1}\mathbf{A}\mathbf{A}^T+\mathbf{I}\end{bmatrix} \\ &= \begin{bmatrix}\frac{1}{\lambda_1}\mathbf{I}+\frac{\lambda_2}{\lambda_1^2}\mathbf{A}^T\mathbf{A} & \frac{1+\lambda_2}{\lambda_1}\mathbf{A}^T+\frac{\lambda_2}{\lambda_1^2}\mathbf{A}^T\mathbf{A}\mathbf{A}^T \\ \frac{\lambda_2}{\lambda_1^2}\mathbf{A}\mathbf{A}^\mathbf{T}\mathbf{A}+\frac{1+\lambda_2}{\lambda_1}\mathbf{A} & \frac{\lambda_2}{\lambda_1^2}\mathbf{A}\mathbf{A}^T\mathbf{A}\mathbf{A}^T+\frac{1+2\lambda_2}{\lambda_1}\mathbf{A}\mathbf{A}^T+(1+\lambda_2)\mathbf{I}\end{bmatrix}\end{aligned} \tag{18}$$

Therefore, by substituting Eq 18 into Eq 16, we get the linear system:

$$\Big(\frac{\lambda_2}{\lambda_1^2}\mathbf{A}\mathbf{A}^T\mathbf{A}\mathbf{A}^T+\frac{1+2\lambda_2}{\lambda_1}\mathbf{A}\mathbf{A}^T+(1+\lambda_2)\mathbf{I}\Big)\alpha = \lambda_2\Big(\frac{1}{\lambda_1}\mathbf{A}\mathbf{A}^T+\mathbf{I}\Big)\mathbf{y}_0 \tag{19}$$

The previous problem can be efficiently diagonalized and solved as follows:

$$\begin{aligned}&\mathbf{F}\Big(\frac{\lambda_2}{\lambda_1^2}diag(\hat{\mathbf{a}}_1\odot\hat{\mathbf{a}}_1^*\odot\hat{\mathbf{a}}_1\odot\hat{\mathbf{a}}_1^*)+\frac{1+2\lambda_2}{\lambda_1}diag(\hat{\mathbf{a}}_1\odot\hat{\mathbf{a}}_1^*)+(1+\lambda_2)\mathbf{I}\Big)\hat{\alpha}^* \\ &=\mathbf{F}\Big(\frac{\lambda_2}{\lambda_1}diag(\hat{\mathbf{a}}_1\odot\hat{\mathbf{a}}_1^*)+\lambda_2\mathbf{I}\Big)\hat{\mathbf{y}}_o^* \\ &\hat{\alpha}^* = \frac{\Big(\frac{\lambda_2}{\lambda_1}(\hat{\mathbf{a}}_1\odot\hat{\mathbf{a}}_1^*)+\lambda_2\Big)\odot\hat{\mathbf{y}}_o^*}{\frac{\lambda_2}{\lambda_1^2}(\hat{\mathbf{a}}_1\odot\hat{\mathbf{a}}_1^*\odot\hat{\mathbf{a}}_1\odot\hat{\mathbf{a}}_1^*)+\frac{1+2\lambda_2}{\lambda_1}(\hat{\mathbf{a}}_1\odot\hat{\mathbf{a}}_1^*)+(1+\lambda_2)} \\ &\hat{\alpha} = \frac{\Big(\frac{\lambda_2}{\lambda_1}(\hat{\mathbf{a}}_1\odot\hat{\mathbf{a}}_1^*)+\lambda_2\Big)\odot\hat{\mathbf{y}}_o}{\frac{\lambda_2}{\lambda_1^2}(\hat{\mathbf{a}}_1\odot\hat{\mathbf{a}}_1^*\odot\hat{\mathbf{a}}_1\odot\hat{\mathbf{a}}_1^*)+\frac{1+2\lambda_2}{\lambda_1}(\hat{\mathbf{a}}_1\odot\hat{\mathbf{a}}_1^*)+(1+\lambda_2)}\end{aligned} \tag{20}$$

Detection Formula with a Single Template

$$\begin{aligned}\mathbf{T}(\mathbf{u}) &= \mathbf{U}\mathbf{w} = \mathbf{U}\mathbf{E}\mathbf{K}^{-1}\mathbf{D}^T\alpha = \mathbf{U}\begin{bmatrix}\mathbf{I}\ \mathbf{0}\end{bmatrix}\begin{bmatrix}\frac{1}{\lambda_1}\mathbf{I} & \frac{1}{\lambda_1}\mathbf{A}^T \\ \frac{1}{\lambda_1}\mathbf{A} & \frac{1}{\lambda_1}\mathbf{A}\mathbf{A}^T+\mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{0} \\ \alpha\end{bmatrix} = \frac{1}{\lambda_1}\mathbf{U}\mathbf{A}^T\alpha \\ &= \frac{1}{\lambda_1}\mathbf{F}diag(\hat{\mathbf{u}}\odot\hat{\mathbf{a}}_1^*)\hat{\alpha}^* \\ \hat{\mathbf{T}}(\mathbf{u}) &= \frac{1}{\lambda_1}\hat{\mathbf{u}}^*\odot\hat{\mathbf{a}}_1\odot\hat{\alpha}\end{aligned} \tag{21}$$

2.2 Using Multiple Templates

For multiple templates, the only difference is that $\tilde{\mathbf{G}}^T = \begin{bmatrix} \mathbf{A}_1^T & \mathbf{A}_2^T & \dots & \mathbf{A}_k^T \\ -\mathbf{I} & -\mathbf{I} & \dots & -\mathbf{I} \end{bmatrix}$ Therefore, the new dual objective is given as follows:

$$\mathbf{D}\tilde{\mathbf{K}}^{-1}\Big(\lambda_2\mathbf{D}^T\mathbf{D} + \lambda_1\mathbf{E}^T\mathbf{E} + \tilde{\mathbf{G}^T}\tilde{\mathbf{G}}\Big)\tilde{\mathbf{K}}^{-1}\mathbf{D}^T\alpha = \lambda_2\mathbf{D}\tilde{\mathbf{K}}^{-1}\mathbf{D}^T\mathbf{y}_0 \tag{22}$$

where

$$\tilde{\mathbf{K}} = \begin{bmatrix} (\sum_i^k \mathbf{A}_i^T\mathbf{A}_i + \lambda_1\mathbf{I}) & -\sum_i^k {\mathbf{A}_i}^T \\ -\sum_i^k \mathbf{A}_i & k\mathbf{I} \end{bmatrix} \tag{23}$$

$$\tilde{\mathbf{K}} = \begin{bmatrix} \mathbf{F} & \mathbf{0} \\ \mathbf{0} & \mathbf{F} \end{bmatrix} \begin{bmatrix} (\sum_i^k diag(\hat{\mathbf{a}}_{1i}^* \odot \hat{\mathbf{a}}_{1i}) + \lambda_1\mathbf{I}) & -\sum_i^k diag(\hat{\mathbf{a}}_{1i}^*) \\ -\sum_i^k diag(\hat{\mathbf{a}}_{1i}) & k\mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{F}^H & \mathbf{0} \\ \mathbf{0} & \mathbf{F}^H \end{bmatrix} \tag{24}$$

To find $\tilde{\mathbf{K}}$, we use the inverse lemma in Eq 3 again. First, we need to find: $\mathbf{B} - \mathbf{N}\mathbf{C}^{-1}\mathbf{V}$:

$$\begin{aligned}
\mathbf{B} - \mathbf{N}\mathbf{C}^{-1}\mathbf{V} &= \sum_i^k diag(\hat{\mathbf{a}}_{1i}^* \odot \hat{\mathbf{a}}_{1i}) + \lambda_1\mathbf{I} - \frac{1}{k}\sum_i^k diag(\hat{\mathbf{a}}_{1i}^*)\sum_i^k diag(\hat{\mathbf{a}}_{1i}) \\
&= diag(\sum_i^k \hat{\mathbf{a}}_{1i}^* \odot \hat{\mathbf{a}}_{1i}) + \lambda_1\mathbf{I} - \frac{1}{k}diag(\sum_i^k \hat{\mathbf{a}}_{1i}^*)diag(\sum_i^k \hat{\mathbf{a}}_{1i}) \\
&= diag(\sum_i^k(\hat{\mathbf{a}}_{1i}^* \odot \hat{\mathbf{a}}_{1i}) + \lambda_1) - \frac{1}{k}diag(\sum_i^k \hat{\mathbf{a}}_{1i}^* \odot \sum_i^k \hat{\mathbf{a}}_{1i}) \\
&= diag\Big(\sum_i^k(\hat{\mathbf{a}}_{1i}^* \odot \hat{\mathbf{a}}_{1i}) + \lambda_1 - \frac{1}{k}\Big(\sum_i^k \hat{\mathbf{a}}_{1i}^* \odot \sum_i^k \hat{\mathbf{a}}_{1i}\Big)\Big)
\end{aligned}$$

Then, using the inverse lemma 3 and similar tricks as used for training the filter in the single template case in the primal domain, the inverse of $\tilde{\mathbf{K}}$ is given as follows:

$$\tilde{\mathbf{K}}^{-1} = \begin{bmatrix} \mathbf{F} & \mathbf{0} \\ \mathbf{0} & \mathbf{F} \end{bmatrix} \Lambda \begin{bmatrix} \mathbf{F}^H & \mathbf{0} \\ \mathbf{0} & \mathbf{F}^H \end{bmatrix}$$

where

$$\Lambda = \begin{pmatrix} diag\Big(\frac{1}{\sum_i^k(\hat{\mathbf{a}}_{1i}^*\odot\hat{\mathbf{a}}_{1i})+\lambda_1-\frac{1}{k}(\sum_i^k\hat{\mathbf{a}}_{1i}^*\odot\sum_i^k\hat{\mathbf{a}}_{1i})}\Big) & diag\Big(\frac{\frac{1}{k}\sum_i^k\hat{\mathbf{a}}_{1i}^*}{\sum_i^k(\hat{\mathbf{a}}_{1i}^*\odot\hat{\mathbf{a}}_{1i})+\lambda_1-\frac{1}{k}(\sum_i^k\hat{\mathbf{a}}_{1i}^*\odot\sum_i^k\hat{\mathbf{a}}_{1i})}\Big) \\ diag\Big(\frac{\frac{1}{k}\sum_i^k\hat{\mathbf{a}}_{1i}}{\sum_i^k(\hat{\mathbf{a}}_{1i}^*\odot\hat{\mathbf{a}}_{1i})+\lambda_1-\frac{1}{k}(\sum_i^k\hat{\mathbf{a}}_{1i}^*\odot\sum_i^k\hat{\mathbf{a}}_{1i})}\Big) & diag\Big(\frac{\frac{1}{k^2}\sum_i^k\hat{\mathbf{a}}_{1i}^*\odot\sum_i^k\hat{\mathbf{a}}_{1i}}{\sum_i^k(\hat{\mathbf{a}}_{1i}^*\odot\hat{\mathbf{a}}_{1i})+\lambda_1-\frac{1}{k}(\sum_i^k\hat{\mathbf{a}}_{1i}^*\odot\sum_i^k\hat{\mathbf{a}}_{1i})}+\frac{1}{k}\Big) \end{pmatrix} \tag{25}$$

And since:

$$\tilde{\Psi} = (\lambda_2 \mathbf{D}^T\mathbf{D} + \lambda_1 \mathbf{E}^T\mathbf{E} + \tilde{\mathbf{G}}^T\tilde{\mathbf{G}}) = \begin{bmatrix} (\sum_i^k \mathbf{A}_i^T\mathbf{A}_i + \lambda_1\mathbf{I}) & -\sum_i^k \mathbf{A}_i{}^T \\ -\sum_i^k \mathbf{A}_i & (k+\lambda_2)\mathbf{I} \end{bmatrix} = \begin{bmatrix} \mathbf{F} & \mathbf{0} \\ \mathbf{0} & \mathbf{F} \end{bmatrix} \Omega \begin{bmatrix} \mathbf{F}^H & \mathbf{0} \\ \mathbf{0} & \mathbf{F}^H \end{bmatrix}$$

where

$$\Omega = \begin{bmatrix} \sum_i^k diag(\hat{\mathbf{a}}_{1i}^* \odot \hat{\mathbf{a}}_{1i}) + \lambda_1\mathbf{I} & -\sum_i^k diag(\hat{\mathbf{a}}_{1i}^*) \\ -\sum_i^k diag(\hat{\mathbf{a}}_{1i}) & (k+\lambda_2)\mathbf{I} \end{bmatrix}$$

Then, $\tilde{\mathbf{K}}^{-1}\tilde{\Psi}\tilde{\mathbf{K}}^{-1} = \begin{bmatrix} \mathbf{F} & \mathbf{0} \\ \mathbf{0} & \mathbf{F} \end{bmatrix} \Lambda\Omega\Lambda \begin{bmatrix} \mathbf{F}^H & \mathbf{0} \\ \mathbf{0} & \mathbf{F}^H \end{bmatrix}$

Then:

$$\Lambda\Omega = \begin{bmatrix} diag(\mathbf{1}) & diag\left(\frac{\frac{\lambda_2}{k}\sum_i^k \hat{\mathbf{a}}_{1i}^*}{\sum_i^k(\hat{\mathbf{a}}_{1i}^*\odot\hat{\mathbf{a}}_{1i})+\lambda_1-\frac{1}{k}(\sum_i^k \hat{\mathbf{a}}_{1i}^*)\odot(\sum_i^k \hat{\mathbf{a}}_{1i})}\right) \\ diag(\mathbf{0}) & diag\left(\frac{\frac{-1}{k}\sum_i^k(\hat{\mathbf{a}}_{1i}^*\odot\hat{\mathbf{a}}_{1i})+\frac{k+\lambda_2}{k}(\sum_i^k \hat{\mathbf{a}}_{1i}^*)\odot(\sum_i^k \hat{\mathbf{a}}_{1i})+\frac{\lambda_1(k+\lambda_2)}{k}}{\sum_i^k(\hat{\mathbf{a}}_{1i}^*\odot\hat{\mathbf{a}}_{1i})+\lambda_1-\frac{1}{k}(\sum_i^k \hat{\mathbf{a}}_{1i}^*)\odot(\sum_i^k \hat{\mathbf{a}}_{1i})}\right) \end{bmatrix}$$

Then:

$$\Lambda\Omega\Lambda = \begin{bmatrix} diag(\mathbf{1}) & diag\left(\frac{\frac{\lambda_2}{k}\sum_i^k \hat{\mathbf{a}}_{1i}^*}{\sum_i^k(\hat{\mathbf{a}}_{1i}^*\odot\hat{\mathbf{a}}_{1i})+\lambda_1-\frac{1}{k}(\sum_i^k \hat{\mathbf{a}}_{1i}^*)\odot(\sum_i^k \hat{\mathbf{a}}_{1i})}\right) \\ diag(\mathbf{0}) & diag\left(\frac{\frac{-1}{k}\sum_i^k(\hat{\mathbf{a}}_{1i}^*\odot\hat{\mathbf{a}}_{1i})+\frac{k+\lambda_2}{k}(\sum_i^k \hat{\mathbf{a}}_{1i}^*)\odot(\sum_i^k \hat{\mathbf{a}}_{1i})+\frac{\lambda_1(k+\lambda_2)}{k}}{\sum_i^k(\hat{\mathbf{a}}_{1i}^*\odot\hat{\mathbf{a}}_{1i})+\lambda_1-\frac{1}{k}(\sum_i^k \hat{\mathbf{a}}_{1i}^*)\odot(\sum_i^k \hat{\mathbf{a}}_{1i})}\right) \end{bmatrix}$$

$$= \begin{pmatrix} diag\left(\frac{1}{\sum_i^k(\hat{\mathbf{a}}_{1i}^*\odot\hat{\mathbf{a}}_{1i})+\lambda_1-\frac{1}{k}(\sum_i^k \hat{\mathbf{a}}_{1i}^*)\odot(\sum_i^k \hat{\mathbf{a}}_{1i})}\right) & diag\left(\frac{\frac{1}{k}\sum_i^k \hat{\mathbf{a}}_{1i}^*}{\sum_i^k(\hat{\mathbf{a}}_{1i}^*\odot\hat{\mathbf{a}}_{1i})+\lambda_1-\frac{1}{k}(\sum_i^k \hat{\mathbf{a}}_{1i}^*)\odot(\sum_i^k \hat{\mathbf{a}}_{1i})}\right) \\ diag\left(\frac{\frac{1}{k}\sum_i^k \hat{\mathbf{a}}_{1i}}{\sum_i^k(\hat{\mathbf{a}}_{1i}^*\odot\hat{\mathbf{a}}_{1i})+\lambda_1-\frac{1}{k}(\sum_i^k \hat{\mathbf{a}}_{1i}^*)\odot(\sum_i^k \hat{\mathbf{a}}_{1i})}\right) & diag\left(\frac{\frac{1}{k^2}\sum_i^k \hat{\mathbf{a}}_{1i}^*\odot\sum_i^k \hat{\mathbf{a}}_{1i}}{\sum_i^k(\hat{\mathbf{a}}_{1i}^*\odot\hat{\mathbf{a}}_{1i})+\lambda_1-\frac{1}{k}(\sum_i^k \hat{\mathbf{a}}_{1i}^*)\odot(\sum_i^k \hat{\mathbf{a}}_{1i})}+\frac{1}{k}\right) \end{pmatrix}$$

$$= \begin{bmatrix} \Upsilon_{11} & \Upsilon_{12} \\ \Upsilon_{21} & \Upsilon_{22} \end{bmatrix}$$

Note that to to compute $\mathbf{D}\tilde{\mathbf{K}}^{-1}\tilde{\Psi}\tilde{\mathbf{K}}^{-1}\mathbf{D}^T$, only the last block Υ_{22} is relevant. So, $\mathbf{D}\tilde{\mathbf{K}}^{-1}\Psi\tilde{\mathbf{K}}^{-1}\mathbf{D}^T = \mathbf{F}diag(\Upsilon_{22})\mathbf{F}^H$, where

$$\begin{aligned}\Upsilon_{22} = &\Bigg(\Big(\frac{\frac{\lambda_1}{k}\sum_i^k \hat{\mathbf{a}}_{1i} + \frac{1}{k}(\sum_i^k \hat{\mathbf{a}}_{1i}) \odot (\sum_i^k \hat{\mathbf{a}}_{1i}^* \odot \hat{\mathbf{a}}_{1i}^*) - \frac{1}{k}(\sum_i^k \hat{\mathbf{a}}_{1i}^*) \odot (\sum_i^k \hat{\mathbf{a}}_{1i})^2}{\sum_i^k(\hat{\mathbf{a}}_{1i}^* \odot \hat{\mathbf{a}}_{1i}) + \lambda_1 - \frac{1}{k}(\sum_i^k \hat{\mathbf{a}}_{1i}^*) \odot (\sum_i^k \hat{\mathbf{a}}_{1i})} - \\ &\frac{1}{k}\sum_i^k \hat{\mathbf{a}}_{1i}\Big) \odot \Big(\frac{\frac{1}{k}\sum_i^k \hat{\mathbf{a}}_{1i}^*}{\sum_i^k(\hat{\mathbf{a}}_{1i}^* \odot \hat{\mathbf{a}}_{1i}) + \lambda_1 - \frac{1}{k}(\sum_i^k \hat{\mathbf{a}}_{1i}^*) \odot (\sum_i^k \hat{\mathbf{a}}_{1i})}\Big)\Bigg) + \\ &\Bigg(\Big(\frac{\frac{k+\lambda_2-1}{k}(\sum_i^k \hat{\mathbf{a}}_{1i}^*) \odot (\sum_i^k \hat{\mathbf{a}}_{1i})}{\sum_i^k(\hat{\mathbf{a}}_{1i}^* \odot \hat{\mathbf{a}}_{1i}) + \lambda_1 - \frac{1}{k}(\sum_i^k \hat{\mathbf{a}}_{1i}^*) \odot (\sum_i^k \hat{\mathbf{a}}_{1i})} + \frac{k+\lambda_2}{k}\Big) \\ &\odot \Big(\frac{\frac{1}{k}\sum_i^k \hat{\mathbf{a}}_{1i}^* \odot \sum_i^k \hat{\mathbf{a}}_{1i}}{\sum_i^k(\hat{\mathbf{a}}_{1i}^* \odot \hat{\mathbf{a}}_{1i}) + \lambda_1 - \frac{1}{k}(\sum_i^k \hat{\mathbf{a}}_{1i}^*) \odot (\sum_i^k \hat{\mathbf{a}}_{1i})} + \frac{1}{k}\Big)\Bigg)\end{aligned} \quad (26)$$

With some mathematical manipulation Eq 26 is exactly equivalent to Eq 7 in the paper [1]. As for the right hand side of Eq 22, one need to find:

$$\mathbf{D}\tilde{\mathbf{K}}^{-1}\mathbf{D}^T = \mathbf{F}diag\left(\frac{\frac{1}{k}\left(\sum_i^k \hat{\mathbf{a}}_{1i}^*\right) \odot \left(\sum_i^k \hat{\mathbf{a}}_{1i}\right)}{\sum_i^k(\hat{\mathbf{a}}_{1i}^* \odot \hat{\mathbf{a}}_{1i}) + \lambda_1 - \frac{1}{k}(\sum_i^k \hat{\mathbf{a}}_{1i}^* \odot \sum_i^k \hat{\mathbf{a}}_{1i})} + \frac{1}{k}\right)\mathbf{F}^H \quad (27)$$

Therefore, solution to Problem 22 is computed as follows:

$$\mathbf{F}diag(\Upsilon_{22})\mathbf{F}^H\alpha = \lambda_2\mathbf{F}\left(\frac{\frac{1}{k}\left(\sum_i^k \hat{\mathbf{a}}_{1i}^*\right) \odot \left(\sum_i^k \hat{\mathbf{a}}_{1i}\right)}{\sum_i^k(\hat{\mathbf{a}}_{1i}^* \odot \hat{\mathbf{a}}_{1i}) + \lambda_1 - \frac{1}{k}(\sum_i^k \hat{\mathbf{a}}_{1i}^* \odot \sum_i^k \hat{\mathbf{a}}_{1i})} + \frac{1}{k}\right)\mathbf{F}^H\mathbf{y}_o \quad (28)$$

$$\hat{\alpha}^* = \lambda_2 diag^{-1}(\Upsilon_{22})\left(\frac{\frac{1}{k}\left(\sum_i^k \hat{\mathbf{a}}_{1i}^*\right) \odot \left(\sum_i^k \hat{\mathbf{a}}_{1i}\right) \odot \hat{\mathbf{y}_0}^*}{\sum_i^k(\hat{\mathbf{a}}_{1i}^* \odot \hat{\mathbf{a}}_{1i}) + \lambda_1 - \frac{1}{k}(\sum_i^k \hat{\mathbf{a}}_{1i}^* \odot \sum_i^k \hat{\mathbf{a}}_{1i})} + \frac{\hat{\mathbf{y}_0}^*}{k}\right) \quad (29)$$

Detection Formula with Multiple Templates

$$\begin{aligned}\mathbf{T}(\mathbf{u}) = \mathbf{U}\mathbf{w} = \mathbf{U}\mathbf{E}\tilde{\mathbf{K}}^{-1}\mathbf{D}^T\alpha = \mathbf{F}&\frac{\hat{\mathbf{u}} \odot \frac{1}{k}\sum_i^k \hat{\mathbf{a}}_{1i}^* \odot \hat{\alpha}^*}{\sum_i^k(\hat{\mathbf{a}}_{1i}^* \odot \hat{\mathbf{a}}_{1i}) + \lambda_1 - \frac{1}{k}(\sum_i^k \hat{\mathbf{a}}_{1i}^* \odot \sum_i^k \hat{\mathbf{a}}_{1i})} \\ \hat{\mathbf{T}}(\mathbf{u}) = &\frac{\hat{\mathbf{u}}^* \odot \frac{1}{k}\sum_i^k \hat{\mathbf{a}}_{1i} \odot \hat{\alpha}}{\sum_i^k(\hat{\mathbf{a}}_{1i}^* \odot \hat{\mathbf{a}}_{1i}) + \lambda_1 - \frac{1}{k}(\sum_i^k \hat{\mathbf{a}}_{1i}^* \odot \sum_i^k \hat{\mathbf{a}}_{1i})}\end{aligned} \quad (30)$$

3 Integrating SRDCF [2]

Instead of re-deriving a closed form solution to SRDCF's [2] formulation, one can use alternating optimization, where the filter $\mathbf{w}$ and the target response $\mathbf{y}$ are updated in an iterative fashion keeping one of them fixed at any given time. The target adapted SRDCF [2] objective becomes:

$$\min_{\mathbf{w},\mathbf{y}} \quad ||\mathbf{A}\mathbf{w} - \mathbf{y}||_2^2 + \lambda_1||\mathbf{X}\mathbf{w}||_2^2 + \lambda_2||\mathbf{y} - \mathbf{y}_0||_2^2, \tag{31}$$

where $\mathbf{X}$ is the weight mask over the filter. Using alternating optimization, it can be minimized by solving the next two optimization problems at iteration j.

$$\mathbf{w}^{j+1} = \arg\min_{\mathbf{w}} ||\mathbf{A}\mathbf{w} - \mathbf{y}^j||_2^2 + \lambda_1||\mathbf{X}\mathbf{w}||_2^2 \tag{32}$$

$$\mathbf{y}^{j+1} = \arg\min_{\mathbf{y}} ||\mathbf{A}\mathbf{w}^{j+1} - \mathbf{y}||_2^2 + \lambda_2||\mathbf{y} - \mathbf{y}_0||_2^2 \tag{33}$$

Problem 32 can be solved using the standard SRDCF [2] solution, while Problem 33 has a closed form solution as follows:

$$\mathbf{y}^{j+1} = \frac{1}{1+\lambda_2}\left(\mathbf{A}\mathbf{w}^{j+1} + \lambda_2\mathbf{y}_o\right) \tag{34}$$

To compute $\mathbf{y}^{j+1}$ efficiently in the Fourier domain, we can use the following strategy:

$$\begin{aligned}
\mathbf{y}^{j+1} &= \frac{1}{1+\lambda_2}(\mathbf{F}diag(\hat{\mathbf{a}}_1)\mathbf{F}^H\mathbf{w}^{j+1} + \lambda_2\mathbf{y}_o) && (35)\\
&= \frac{1}{1+\lambda_2}(\mathbf{F}diag(\hat{\mathbf{a}}_1)\hat{\mathbf{w}}^{*j+1} + \lambda_2\mathbf{y}_o) && (36)\\
\Rightarrow \hat{\mathbf{y}}^* &= \frac{1}{1+\lambda_2}(diag(\hat{\mathbf{a}}_1)\hat{\mathbf{w}}^{*j+1} + \lambda_2\hat{\mathbf{y}}_o^*) && (37)\\
&= \frac{1}{1+\lambda_2}(\hat{\mathbf{a}}_1 \odot \hat{\mathbf{w}}^{*j+1} + \lambda_2\hat{\mathbf{y}}_o^*) && (38)
\end{aligned}$$

Acknowledgments. Research in this publication was supported by the King Abdullah University of Science and Technology (KAUST) Office of Sponsored Research.

References

1. Bibi, A., Mueller, M., Ghanem, B.: Target response adaptation for correlation filter tracking. In: European Conference on Computer Vision, ECCV, 2016.
2. Danelljan, M., Hager, G., Shahbaz Khan, F., Felsberg, M.: Learning spatially regularized correlation filters for visual tracking. In: IEEE International Conference on Computer Vision, ICCV, 2015.