Local Color Mapping Combined with Color Transfer for Underwater Image Enhancement - Supplementary Material

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Abstract

Color correction and color transfer methods have gained a lot of attention in the past few years to circumvent color degradation that may occur due to various sources. In this paper, we propose a novel simple yet powerful strategy to enhance profoundly color distorted underwater images. The proposed approach combines both local and global information through a simple yet powerful affine transform model. Local and global information are carried through local color mapping and color covariance mapping between an input and some reference source, respectively. Several experiments on degraded underwater images demonstrate that the proposed method performs favourably to all other methods including ones that are tailored to correcting underwater images by explicit noise modelling.

1. Proposed method’s result gradients computation

For proposed problem:

$$\min_{A, b} \frac{1}{2} \left( \| A - Y + b \|_F^2 + \frac{\lambda_1}{2} \| A C_i A^T - C_r \|_F^2 + \frac{\lambda_2}{2} \| A \|_F^2 + \| b \|_F^2 \right)$$

Local Color Mapping

$$= g_1(A, b) + g_2(A) + g_3(A, b)$$

We solve gradients in given manner:

1.1. Gradient with respect to A

$$g_1(A) = \frac{1}{2} \langle A X - Y + b 1^T, A X - Y + b 1^T \rangle$$

$$\partial g_1 = \langle \partial A X, A X - Y + b 1^T \rangle = \langle \partial A, (A X - Y + b 1^T) X^T \rangle$$

Then:

$$\nabla_A g_1(A) = (A X - Y + b 1^T) X^T$$

As for computing $\nabla_A g_2(A)$, make the following change of variable for simpler notation, $\Psi = A C_i A^T - C_r$; therefore the cost to find the gradient over is $g_2 = \frac{\lambda_2}{2} \langle \Psi, \Psi \rangle$.

$$\partial g_2 = \lambda_1 \langle \Psi, \partial \Psi \rangle = \lambda_1 \langle \Psi, \partial \left( A C_i A^T - C_r \right) \rangle$$

$$= \lambda_1 \langle \Psi, \partial \left( A C_i A^T \right) \rangle = \lambda_1 \langle \Psi, A C_i \partial (A^T) \rangle$$

$$\partial g_2 = \lambda_1 \langle \partial A, \Psi A C_i^T \rangle + \lambda_1 \langle \partial (A C_i A^T), \Psi \rangle$$

$$= \lambda_1 \langle \partial A, \Psi A C_i^T \rangle + \lambda_1 \langle \partial (A) C_i A^T, \Psi \rangle$$

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Since all covariances matrices are symmetric that is $C_i^T = C_i$, and $C_r^T = C_r$. Then:

$$\nabla_A g_2(A) = \lambda_1 (2 A C_i A^T C_i - 2 C_r A C_i)$$

$$= 2 \lambda_1 A C_i A^T C_i - 2 \lambda_1 C_r A C_i$$

As for lastly $\nabla_A g_3 = 2 A$. Therefore the final gradient with respect to A is as follows:

$$\nabla_A f(A) = \nabla_A g_1(A) + \nabla_A g_2(A) + \nabla_A g_3(A)$$

$$= \lambda_1 (A X - Y + b 1^T) X^T + 2 \lambda_1 A C_i A^T A C_i - 2 \lambda_1 C_r A C_i + 2 A$$
1.2. Gradient with respect to $b$

\[ g_1(b) = \frac{\lambda_1}{2} (AX - Y + b_1^T, AX - Y + b_1^T) \]
\[ \partial g_1 = \lambda_1 (\partial b_1^T, AX - Y + b_1^T) \]
\[ = \lambda_1 (\partial b_1 (AX - Y + b_1^T)1) \]  

(7)

Then:

\[ \nabla_b g_1(b) = \lambda_1 (AX - Y + b_1^T)1 \]  

(8)

Since $\nabla_b g_2 = 0$, and $\nabla_b g_3 = \lambda_2 b$; therefore the final gradient with respect to $A$ is as follows:

\[ \nabla_b f(b) = \nabla_b g_1(b) + \nabla_b g_3(b) \]
\[ = \lambda_1 (AX - Y + b_1^T)1 + \lambda_2 b \]  

(9)